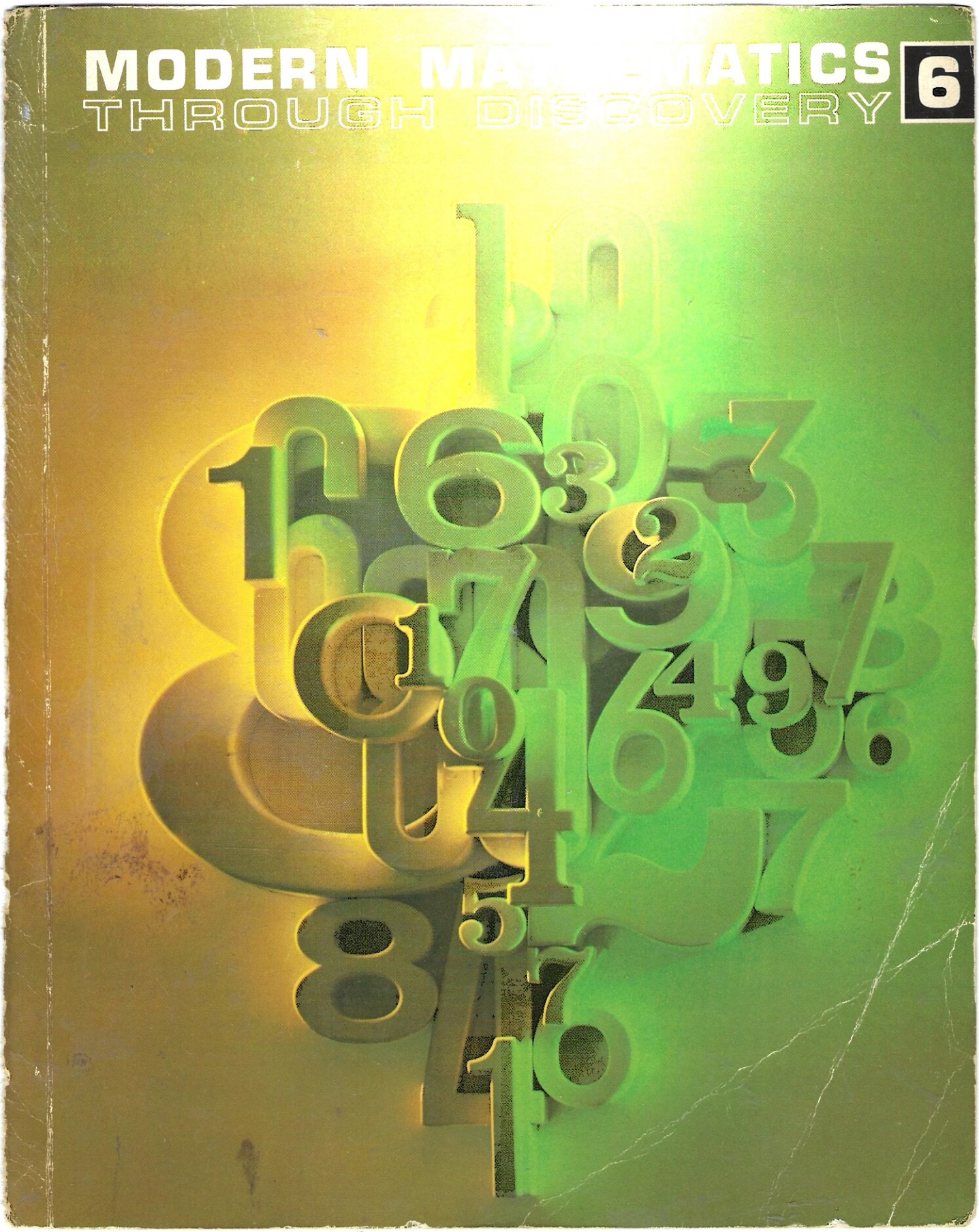
MODERN MATHEMATICS THROUGH DISCOVERY 6



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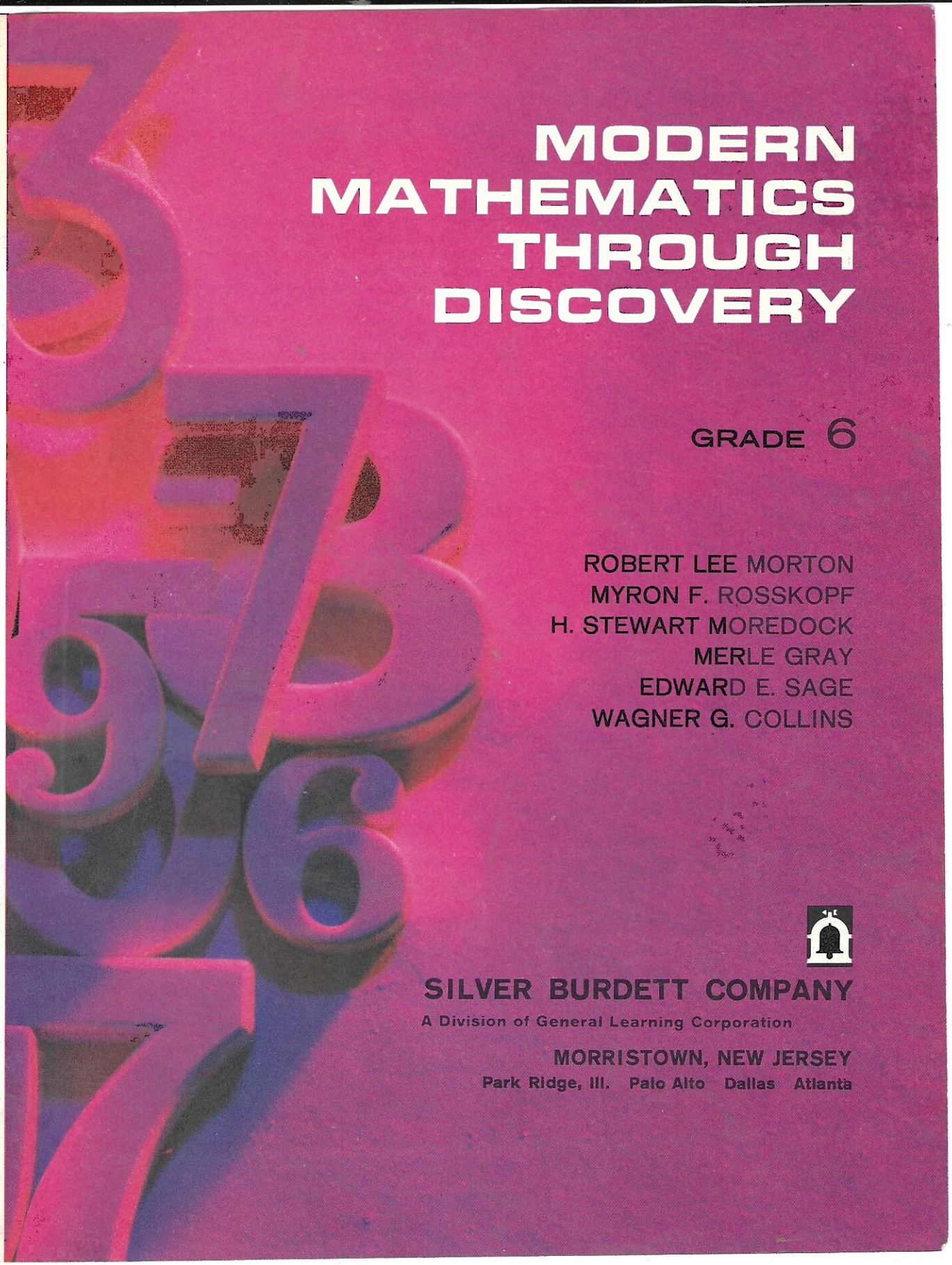

MODERN MATHEMATICS THROUGH DISCOVERY

GRADE 6

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PREFACE

IN SCIENCE, in industry, in commerce—indeed in every area of our complex society—mathematics plays a role of ever-increasing importance. To provide our children with the necessary foundation for their future, leaders in the field of mathematics education have long seen the need to revitalize the mathematics curriculum; they have sought a new program that clarifies mathematical ideas, offers stimulating content, and helps students prepare for intelligent participation in a society that will be even more technological than our own.

MODERN MATHEMATICS THROUGH DISCOVERY is such a program. Modern in viewpoint, in content, and in spirit, it is a sequential learning program in which new approaches and new content are always utilized for a valid mathematical and pedagogical purpose.

In their preparation of this series of books, the authors examined carefully several exploratory programs in elementary mathematics. These included programs of the following: the School Mathematics Study Group, the Madison Project, the University of Illinois Committee on School Mathematics, the University of Maryland Mathematics Project, the Greater Cleveland Mathematics Program, and others. Each of these sources supplied useful ideas.

Throughout their work, the authors were guided by a concern for the learner and a consciousness of the teacher's problems. The material was then presented with a careful craftsmanship based on years of authorship and teaching experience.

The success of the series in thousands of schools has proved the wisdom of the principles on which this program was based. Building on this success in the classroom, the authors have expanded the mathematical content of the series, while maintaining the careful craftsmanship and concern for the learner that distinguish their work.

It is the strong conviction of the authors that modern mathematics should not be regarded solely as "mathematics for the better pupil." Through a heightened emphasis on meaning and through careful attention to *appropriate* learning experiences, pupils of average or less-than-average ability can benefit as much as superior pupils. MODERN MATHEMATICS THROUGH DISCOVERY is constructed to make possible, for the average

class, a meaningful course in modern mathematics—a course that will best serve the future interests of the majority of pupils.

One significant aspect of the series is its consistent, sustained use of number sentences (equations and inequalities) to illuminate ideas about numbers and operations. The number line is also utilized, both as a graphic tool and as a concept, to build important basic understandings. The text carefully introduces and maintains the idea of addition and subtraction, and multiplication and division, as inverse operations. Strong emphasis is given to ideas of place value in our decimal system, and the pupils' understanding of the system is broadened—as early as the fifth and sixth grades—by work with other place-value numeration systems. Informally, ideas of structure are introduced and utilized; then, grade by grade, the ideas are given heightened emphasis. The basic concepts and notation of sets are used to unify and clarify various topics in the course. Throughout the grades, the pupils are encouraged to look for patterns and relationships among numbers.

To be truly modern, a mathematics program must be modern in spirit. It must be a program that emphasizes inquiry, exploration, and discovery. MODERN MATHEMATICS THROUGH DISCOVERY is constructed to stimulate this inquiry and this discovery. The texts carefully help pupils to move—step by step—from one idea to another; and throughout this procedure, the pupils are encouraged to explore and discover relationships. The classroom teacher has, of course, always encouraged exploration of ideas by the pupil. Now, with the aid of texts that consciously promote this exploration, deeper understandings will result.

MODERN MATHEMATICS THROUGH DISCOVERY offers—in effective, practical textbook form—a proven elementary school program that will prepare pupils for their later study of mathematics and will communicate the excitement of ideas.

After a school play, the director took a photograph of the boys and girls in the play.

1. Think of three *sets* in the photograph—the set of girls, the set of boys, and the set of chairs. Without counting, answer these questions.

a. Are there as many chairs as girls? How do you know?

b. Are there more chairs than boys, or are there more boys than chairs?

2. A girl is sitting in each chair. Are there any chairs left over? Are there any girls left over? If you changed the seating arrangement and each girl were in a different chair, would there be any chairs left over? any girls left over? Is there an exact matching between the *members* of the set of girls and the members of the set of chairs?

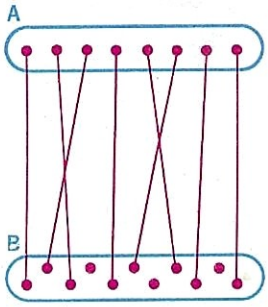
▶ When you can match, or pair, each member of one set with exactly one member of a second set, and neither set has any members left over, then these sets have the same **cardinal number.** We call this number the ***number of the set.***

3. The cardinal number of a set tells how many members the set has. What is the cardinal number of the set of chairs? the cardinal number of the set of girls? Does the numeral "7" name this number? What Roman numeral is a name for the same number?

Since VII and 7 are names for the same number, is this sentence true: VII=7?

4. What is the cardinal number of the set of boys in the photograph? Write a true sentence that shows two names for this number.

2 COMPARING NUMBERS



1. This drawing shows two sets, A and B. Lines have been drawn to match each member of set A with a member of set B. Which set has members left over? Does set B have more members than set A?

If one set has more members than another set, we say the cardinal number of the first set is **greater than** the cardinal number of the second set.

2. What is the cardinal number of set A? of set B? Is 11 greater than 8? Using the sign $>$ for "is greater than," we write this true sentence: $\mathbf{11 > 8}$.

3. Can we say that the cardinal number of set A **is less than** the cardinal number of set B? Then we can also write this true sentence: $\mathbf{8 < 11}$. The sign $<$ means "is less than."

4. Read each of the following.

a. $6 < 13$ **b.** $9 > 3$ **c.** $0 < 1$ **d.** $73 > 69$

Is each sentence in **4a–d** true? You know that some number sentences are false. Which of the following are true? Which are false?

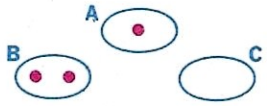
	a	b	c	d
5.	$26 > 25$	$25 > 26$	$25 < 26$	$26 < 25$
6.	$83 < 80$	$83 > 80$	$80 < 83$	$80 > 83$
7.	$99 < 100$	$175 < 150$	$\text{VI} > \text{IV}$	$\text{X} < \text{VIII}$
8.	$909 < 990$	$899 < 989$	$9099 > 9100$	$100{,}000 > 99{,}999$
9.	$\text{XXIII} = 23$	$\text{IX} = 11$	$\text{XI} = 11$	$\text{XI} > 10$

Copy, and put in $>$, $<$, or $=$ in place of $\square$ to make the sentence true.

10.	$85 \square 75$	$200 \square 165$	$385 \square 295$	$1 \square 0$
11.	$\text{XX} \square 19$	$\text{XX} \square 20$	$\text{XX} \square 21$	$\text{XXI} \square 21$
12.	$950 \square 1000$	$950 \square 900$	$7035 \square 7305$	$8100 \square 8001$

1. How many members does set A have? set B? set C?

Is set C empty? We call it the **empty set**. Its cardinal number is 0.



2. Draw a set R that has one more member than set B. What is the cardinal number of R?

3. Draw a set Q that has one more member than set R. What is the cardinal number of set Q?

4. Suppose there were 2250 people in a theater. If one more person came in, what would be the cardinal number of the set of people in the theater?

5. The smallest cardinal number is 0. Do you think there is a greatest cardinal number?

6. Begin with the number 0 and name the cardinal numbers, in order, from 0 through 9. Think of this set of numbers as set T.

7. Does the following show the members of set T in the order you named them?

$$T = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

The *braces*, { }, are used to show that we are thinking of the collection as a set.

Here are some other sets of cardinal numbers. Is every member of {6, 7, 8} a member of set T? Then we say that {6, 7, 8} is a **subset** of T.

$\{6, 7, 8\}$

$\{2, 3\}$

$\{9, 10, 11\}$

8. Is every member of {2, 3} a member of T? Then is {2, 3} a subset of T?

Is every member of {9, 10, 11} a member of T? Then {9, 10, 11} *is not* a subset of T.

9. Tell which of the following *are not* subsets of T.

a. {0, 1} **b.** $\{\frac{1}{2}, 1\}$ **c.** {2} **d.** {20, 21} **e.** {8, 9, 10}

$T = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

4 PARTITIONS OF SETS

1. Study these two sets.

$\{0, 2, 4, 6, 8\}$ $\{1, 3, 5, 7, 9\}$

a. Can we describe the first set as the set of even numbers belonging to T? Describe the second set.

b. Is each set a subset of T?

c. Do the two sets have any members in common? That is, is there a member of one set that is a member of the other set?

d. If you join the two sets, do you get set T?

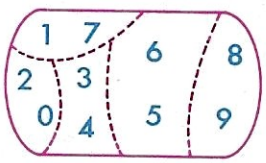
▶ We say that the sets $\{0, 2, 4, 6, 8\}$ and $\{1, 3, 5, 7, 9\}$ are a **partition** of set T because (1) the two sets have no members in common and (2) joining the two sets gives set T.

2. Here are four subsets of T: $\{0, 1, 2\}$, $\{3, 4, 5\}$, $\{6, 7, 8\}$, and $\{9\}$.

Do any of these subsets have members in common? If you join the four sets, do you get set T? Then are these subsets a partition of set T?

3. Study these four subsets of T: $\{0, 1, 2\}$, $\{3, 4, 5\}$, $\{6, 7, 8\}$, and $\{7, 8, 9\}$. Are they a partition of T? Why, or why not?

4. Look at these subsets of T: $\{0, 1, 2, 3\}$ and $\{4, 5, 6, 7\}$. Are they a partition of T? Why, or why not?

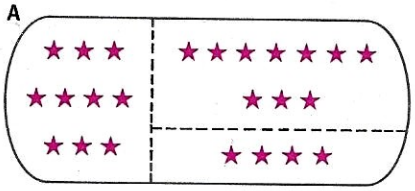


5. This drawing shows a partition of T into subsets with two members in each subset. Can you make a different partition in which each subset has two members? Show it.

6. Partition T, making as many subsets of four members as possible. In such a partition, how many subsets of four are there? How many members are there in the remaining subset?

1. Set A has been partitioned into as many subsets of ten as possible. How many subsets of ten are there? How many stars are there in the subset which does not have ten members?

A



What is the cardinal number of A? Write the numeral.

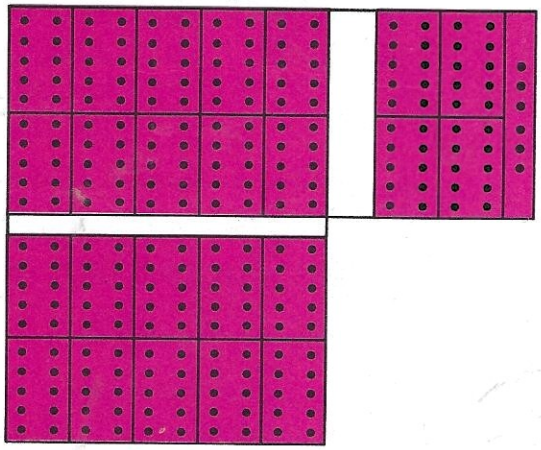
In the numeral 24 which digit tells the number of tens? Which digit tells the number of ones?

2. Study the partition of set B. How many subsets of ten are there? If we group ten subsets of ten together, how many subsets of "ten tens" are there?

What is the cardinal number of set B? Write a three-place numeral to name this number.

In the numeral 246, what does the 2 mean? the 4?

B



3. We call our numeral system the **decimal system**, because we group by tens in order to express numbers. (The word decimal comes from a Latin word meaning "ten.") How many digits do we use to write numerals in our system?

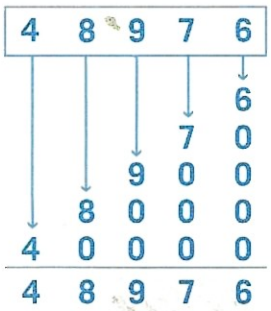
4. The decimal system is a *place-value* numeral system. The place of a digit in a numeral has meaning—a value is assigned to each place. In the numeral 2487, which digit is in ones' place? Which digit is in tens' place? in hundreds' place? in thousands' place? The name of each place suggests the value given to that place. In 2487, what does the 2 mean? the 4?

5. Tell what the 2 and the 4 mean in:

a. 4253 **b.** 24,685 **c.** 42,399 **d.** 245,683

1. The paid attendance at a baseball game was 48,976. This numeral tells the total number of tickets collected.

As this diagram shows, each digit in the numeral 48,976 has a certain value.



The 6 represents 6 ones.
The 7 represents 7 tens, or 70.
The 9 represents 9 hundreds, or 900.
The 8 represents 8 thousands, or 8,000.
The 4 represents 4 ten thousands, or 40,000. Read the sum. Is the number named the sum of the values of the digits?

2. Make a similar diagram to explain this numeral: 51,632.

3. Tell what each digit represents in the following:

a. 7921 **b.** 34,583 **c.** 123,604 **d.** 4,394,540

4. To make the reading of large numerals easy, the digits are grouped in sets of three, called **periods**, beginning at the right. Commas are used to separate the periods. The words at the top of this chart name some of the periods.

billions			millions			thousands			units		
hundred	ten	one	hundred	ten	one	hundred	ten	one	hundreds	tens	ones
8	1	5,	7	9	2,	0	4	3,	6	5	3

When we read the numeral above, we use the names of all the periods except the units' period: "eight hundred fifteen *billion*, seven hundred ninety-two *million*, forty-three *thousand*, six hundred fifty-three." Read the following.

a. 56,783 **b.** 120,079 **c.** 33,576,854

d. 1,456,975,008 **e.** 492,116,320 **f.** 781,003,958,362

1. Are the following statements true of the numeral 4352?

a. The 4 represents 4000, or 4×1000.

b. The 3 represents 300, or 3×100.

c. The 5 represents 50, or 5×10.

d. The 2 represents 2×1.

What is the missing numeral in this sentence?

$$N = (4 \times 1000) + (3 \times 100) + (5 \times 10) + (2 \times 1)$$

2. Write the missing numeral.

a. $N = (7 \times 1000) + (8 \times 100) + (6 \times 10) + (5 \times 1)$

b. $N = (8 \times 10{,}000) + (9 \times 1000) + (9 \times 100) + (6 \times 10) + (5 \times 1)$

3. What is 10×1? Which place in a decimal numeral has the value 10?

What is 10×10? Which place in a decimal numeral has the value 100?

What is $10 \times 10 \times 10$? Which place in a decimal numeral has the value 1000?

In every decimal numeral, is the value of a place ten times that of the place immediately at its right?

4. The table below shows the value of each digit in 935,627. Copy the table and complete it.

Place	Digit	Value	Value equals
ones'	7	7×1	?
tens'	2	2×10	?
hundreds'	6	$6 \times 10 \times 10$	?
thousands'	5	$5 \times 10 \times 10 \times 10$	?
ten-thousands'	3	$3 \times 10 \times 10 \times 10 \times 10$	?
hundred-thousands'	9	$9 \times 10 \times 10 \times 10 \times 10 \times 10$	?
		Total	?

5. For each of the following numerals, express the value of each digit, and the total, as in the last column of Exercise **4**.

a. 8967 **b.** 49,715 **c.** 507,003 **d.** 3,802,567

8

EXPONENTS

1. You have seen that 100 can be expressed as a product of tens: $100 = 10 \times 10$. Express 1000 as a product of tens. Express 10,000 as a product of tens. How many tens are used to express 100,000 as a product of tens?

2. There is a short way to show a product of 10's. We can write 10^2 for 10×10; 10^3 for $10 \times 10 \times 10$. What can we write for $10 \times 10 \times 10 \times 10$? What does 10^5 mean?

3. You can see that we are using 10's as *factors* in these products. How many times is 10 used as a factor in 10^2? in 10^3? in 10^4?

4. In an expression such as 10^4, the "4" is called the **exponent**. The "10" is called the base. What is the exponent in 10^2? 10^3? 10^6? What is the base in each case?

▶ The exponent tells how many times the base is to be used as a factor in a product.

We read an expression in exponent form as follows:
10^2 is read "ten to the second power," or "ten squared"
10^3 is read "ten to the third power," or "ten cubed"
10^4 is read "ten to the fourth power"
10^7 is read "ten to the seventh power"

5. Study the sentences at the right. What is the relation between the exponent of 10 and the number of zeros in the corresponding decimal numeral? Use this pattern to write a decimal numeral for each of the following.

$10^2 = 100$
$10^3 = 1000$
$10^4 = 10{,}000$
$10^5 = 100{,}000$

a. 10^8 **b.** 10^6 **c.** 10^9 **d.** 10^7

6. You know that $400 = 4 \times 100$. Then what does 4×10^2 equal? Copy each of the following, replacing N with the decimal numeral that will make a true sentence.

a. $5 \times 10^2 = N$ **b.** $3 \times 10^4 = N$ **c.** $7 \times 10^3 = N$

7. Replace N with the correct power of 10.

a. $600 = 6 \times N$ **b.** $9000 = 9 \times N$ **c.** $20{,}000 = 2 \times N$

1. Express each of the following as a single digit times 10 or a power of 10.

a. 70 **b.** 30,000 **c.** 300 **d.** 9000 **e.** 10,000

2. Study the meaning of 3856 shown below.

$$\begin{aligned} 3856 &= 3000 + 800 + 50 + 6 \\ &= (3 \times 1000) + (8 \times 100) + (5 \times 10) + (6 \times 1) \\ &= (3 \times 10^3) + (8 \times 10^2) + (5 \times 10) + (6 \times 1) \end{aligned}$$

The three rows above show different ways to express a numeral in **expanded form**. Is the place value of each digit shown? Is the meaning of the number expressed by using 1, 10, and powers of 10?

3. Copy the expanded forms below and replace each ? with a power of 10, with 10, or with 1.

a. $743 = (7 \times ?) + (4 \times ?) + (3 \times ?)$

b. $2{,}639 = (2 \times ?) + (6 \times ?) + (3 \times ?) + (9 \times ?)$

c. $65{,}083 = (6 \times ?) + (5 \times ?) + (0 \times ?) + (8 \times ?) + (3 \times ?)$

4. Express each of the following in expanded form.

a. 894 **b.** 1328 **c.** 47,823 **d.** 1,056,794

5. Exponents can be used with bases other than 10. For example, 3^3 means $3 \times 3 \times 3$. What does 4^2 mean? Are "4^2," "4×4," and "16" all names for the same number?

6. The sentence at the right indicates that a product of 5's and a power of 5 name the same number. Write a similar sentence for each of the following.

$$5 \times 5 \times 5 = 5^3$$

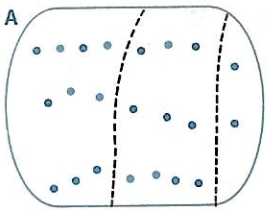
a. 2×2 **b.** $9 \times 9 \times 9$ **c.** 13×13 **d.** $11 \times 11 \times 11 \times 11$

e. 5^4 **f.** 3^6 **g.** 2^7 **h.** 6^3

7. Copy each of the following sentences, replacing N with the proper decimal numeral.

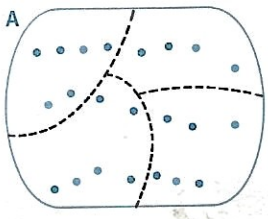
a. $6^2 = N$ **b.** $6^3 = N$ **c.** $6^4 = N$

10 ANOTHER NUMERAL SYSTEM



1. The decimal system is sometimes called the *base ten* system. Can you tell why?

This drawing shows a set partitioned into as many subsets of ten as possible. Does "2 tens and 2 ones" tell the number of set *A*? What is the decimal, or base ten, numeral for this number?

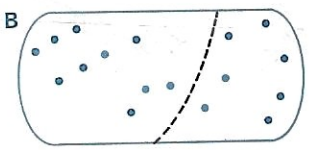


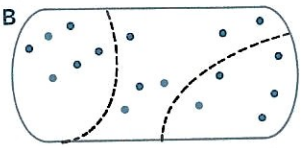
2. We can use another number as the base of a numeral system. Here the same set *A* has been partitioned into as many subsets of six as possible. How many subsets of six are there? How many dots are there in the remaining subset?

Does "3 sixes and 4 ones" tell the number of the set? In a numeral system based on six, we write the numeral 34_{six} to name this number. The *six* tells us that "six" is the base of the system. We read 34_{six} as "three, four, base six."

3. Is this sentence true: $22 = 34_{six}$? How do you know?

4. In these drawings, a set has again been partitioned in two different ways.



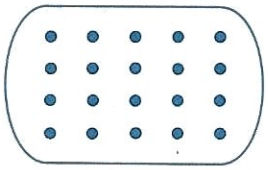


a. What is the base ten numeral for the number of the set?

b. Does "2 sixes and 5 ones" tell the number of the set? Then which of the following is the base six numeral for the number: 52_{six}, or 25_{six}?

c. What is $(2 \times 6) + 5$?

5. Draw a set of 20 dots. Now partition it into as many subsets of six as possible.



a. How many subsets of six are there? How many dots are there in the remaining subset?

b. Copy, and put in the missing numeral: $20 = \square_{six}$.

6. Does 32_{six} mean "3 sixes and 2 ones"? What is $(3 \times 6) + 2$?

7. Draw sets of dots and find the base six numerals for the following.

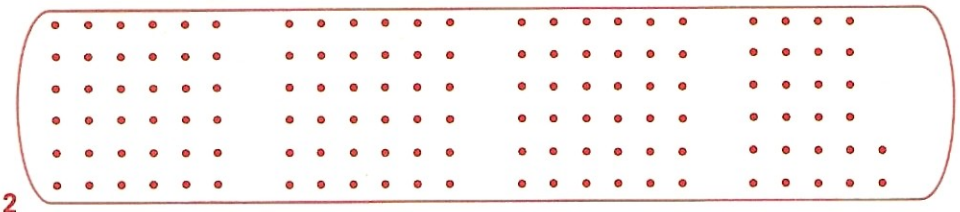
a. 19	**b.** 14	**c.** 25	**d.** 24	**e.** 35	**f.** 30
g. 23	**h.** 13	**i.** 12	**j.** 8	**k.** 7	**l.** 6
m. 33	**n.** 29	**o.** 21	**p.** 10	**q.** 18	**r.** 16

8. In the base ten system, we use the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. When we have 9 ones and 1 more, what numeral do we write in base ten? How many places does it have? What does the 1 mean? the 0?

9. In the base six system, we use the digits 0, 1, 2, 3, 4, 5. When we have 5 ones and 1 more in the base six system, what numeral do you think we write? How many places does it have? What does the 1 in 10_{six} mean? What does the 0 mean? Remember to read 10_{six} as "one, zero, base six."

10. Does the numeral 20_{six} mean "2 sixes and 0 ones"? What does 30_{six} mean? 40_{six}? 50_{six}?

11. Does the numeral 6 in the base ten system name the same number as 10_{six}? What is the base ten numeral that names the same number as 20_{six}? 30_{six}? 40_{six}? 50_{six}?



12

WHAT BASE SIX NUMERALS MEAN

1. Use a base ten numeral to express the number of dots in this set.

In base ten, when we partition a set and have ten subsets of ten, we join them to form a subset with a greater number of members. How many?

2. In the base six numeral system, when we partition a set and have six subsets of six, we join them to form a subset of six sixes. How many members are there in a subset of six sixes?

3. In a base ten numeral, the first place at the right is ones' place; the place at the left of ones' place is tens' place; the next place is hundreds' place (10^2). Does the numeral 134 mean $(1 \times 10^2) + (3 \times 10) + 4$?

4. In a base six numeral, the first place at the right is ones' place; but the place at the left of ones' place is sixes' place; and the next place is thirty-sixes' place (6^2).

In the set of dots at the top of the page, how many subsets of thirty-six are there? how many subsets of six? how many extra dots?

Then to express the number of dots as a base six numeral, we write: 342_{six}. The numeral 342_{six} is read, "three, four, two, base six," and it means $(3 \times 6^2) + (4 \times 6) + 2$.

5. Suppose that there were exactly 4 subsets of thirty-six. Then could the number of dots be expressed as this base six numeral: 400_{six}? How do you read this numeral?

6. Write a base six numeral for each of the following.

a. 2 thirty-sixes, 3 sixes, 4 ones

b. 3 thirty-sixes, 5 sixes, 3 ones

c. 1 thirty-six, 2 sixes, 0 ones

d. 5 thirty-sixes, 0 sixes, 0 ones

7. To express 95 as a base six numeral, you need to know how many subsets of thirty-six there are in a set of 95; how many subsets of six; and how many ones. To find the number of members in the subsets, you can divide.

How many 36's in 95? (2, remainder 23) Write "2" in thirty-sixes' place as shown.

How many 6's in 23? (3, remainder 5) Write "3" in sixes' place.

How many ones are left? (5) Write "5" in ones' place.

Add. Then 95, expressed as a base six numeral, is 235_{six}.

thirty-sixes	sixes	ones
2	0	0
	3	0
		5
2	3	5 $_{six}$

Are 95 and 235_{six} names for the same number?

8. To express 49 as a base six numeral, you can use the same procedure.

First, fill thirty-sixes' place.
(49 ÷ 36 = ?, remainder ?)

Next fill sixes' place.
(13 ÷ 6 = ?, remainder ?)

How many ones will there be in ones' place?

Add, and express the sum as a base six numeral.

thirty-sixes	sixes	ones
?	0	0
	?	0
		?

9. Express as a base six numeral.

a. 45 **b.** 77 **c.** 118 **d.** 127 **e.** 188 **f.** 202

10. If you express 35 as a base six numeral, will there be only two places in the numeral? Why?

Express 35 as a base six numeral; then express 36 as a base six numeral.

If you are counting in base six, is 100_{six} the next numeral after 55_{six}?

11. What is 72 expressed as a base six numeral? 108? 180?

12. Express 215 as a base six numeral.

14

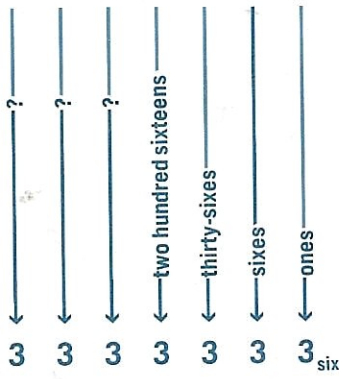
EXPRESSING NUMBERS IN THE BASE SIX SYSTEM

1. In the base six system, we express numbers using one, six, and powers of six. Copy and complete this table of powers of six. It will be useful in working with base six numerals.

$6^2 = 6 \times 6$, or 36
$6^3 = 6 \times 6 \times 6$, or 216
$6^4 = ?$ or ?
$6^5 = ?$ or ?
$6^6 = ?$ or ?

2. The diagram at the right shows place value in the base six system. In this system, each place has 6 times the value of the place at its right.

Copy and complete the chart to seven places.



3. Use the table and the chart to help you change 1761 to a base six numeral.

What is the highest power of six that can be used to divide 1761? Begin to divide, using this number.

a Divide 1761 by 1296 (6^4).

$$1296\overline{)1761}\quad 1$$
$$1296$$
$$465$$

b Divide 465 by 216 (6^3).

$$216\overline{)465}\quad 2$$
$$432$$
$$33$$

c Divide 33 by 36 (6^2).

$$36\overline{)33}\quad 0$$

d Divide 33 by 6.

$$6\overline{)33}\quad 5$$
$$30$$
$$3$$

1 2 0 5 3_{six}

6^4	6^3	6^2	6	1

4. For each number named below, find the base six numeral. Be sure to divide first by the highest power of six you can. Then decide how many places the base six numeral will have. Put a digit in each place; sometimes it will be zero.

a. 493 **b.** 986 **c.** 1925 **d.** 2682

1. To find a base six numeral for 328 or 2809, you divide by powers of six. If you are given a base six numeral such as 5304_{six}, how can you find the base ten numeral?

To find the base ten numeral for 5304_{six}, you multiply the value of each place by the digit in that place, as shown. When you add the products, you get 1192. Is 1192 the base ten numeral equivalent to 5304_{six}? Check by converting 1192 to a base six numeral.

$5\ 3\ 0\ 4_{six}$	Base Ten
$(5 \times 6^3) =$	1080
$(3 \times 6^2) =$	108
$(0 \times 6) =$	0
$(4 \times 1) =$	4
	1192

Are 5304_{six} and 1192 two names for the same number? The first is a base ? name, and the second is a base ? name.

2. Express each of the following as a base ten numeral.

a. 45_{six} **b.** 324_{six} **c.** 1043_{six} **d.** 24034_{six}

3.a. Base ten numerals use ten digits. What are they?

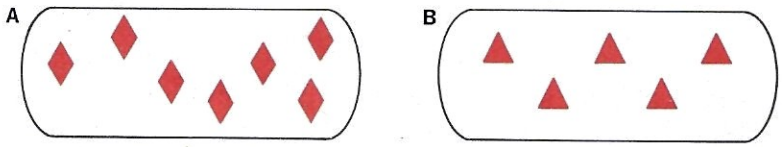
b. Base six numerals use only six digits. What are they?

c. Count from 0_{six} to 101_{six}.

4. You have been working with different numerals for cardinal numbers. Another name for a cardinal number is **whole number**. We can represent the set of whole numbers as follows: {0, 1, 2, 3, · · ·}. What do the three dots mean? Is it also correct to represent the set of whole numbers as $\{0_{six}, 1_{six}, 2_{six}, 3_{six}, \cdots\}$?

Copy, and write >, =, or < in place of ▭ to form a true sentence.

	a	b	c
5.	$7 \square 10_{six}$	$15_{six} \square 12$	$24_{six} \square 16$
6.	$15_{six} \square 15$	$100_{six} \square 60$	$1000_{six} \square 200$
7.	$5000_{six} \square 1080$	$5010_{six} \square 1090$	$5100_{six} \square 1100$



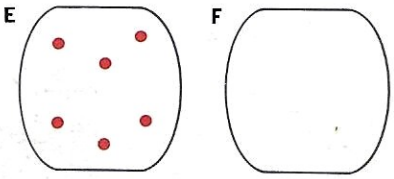
1. Do sets A and B have any members in common? How many members does set A have? set B?

Think of joining sets A and B. Can we add 7 and 5 to find the number of members in the new set?

What is $7 + 5$? $5 + 7$? Is 12 the number of the new set?

2. What is $10 + 8$? $8 + 10$? Is this sentence true: $10 + 8 = 8 + 10$?

▶ If $\square$ and $\triangle$ represent any whole numbers, then $\square + \triangle = \triangle + \square$. This sentence states the **commutative principle of addition.**



3. How many members does set E have? set F? Now think of joining sets E and F. What is $6 + 0$? $0 + 6$?

4. What is $18 + 0$? $0 + 32$?

▶ If N represents any whole number, then $0 + N = N$ and $N + 0 = N$. We call zero the **identity element for addition.**

Find the missing numerals.

5.a. $(7 + 6) + 5 = \square$ **b.** $7 + (6 + 5) = \square$

6.a. $(8 + 3) + 4 = \triangle$ **b.** $8 + (3 + 4) = \triangle$

Compare the sums in **5a** and **b**. Are they the same? Compare the sums in **6a** and **b**. Are they the same?

▶ If $\square$, $\triangle$, and $\diamond$ represent any whole numbers, then $(\square + \triangle) + \diamond = \square + (\triangle + \diamond)$. This sentence states the **associative principle of addition.**

1. Each sentence below tells about adding the same three numbers. Find the sums.

$(8+7)+6=\diamond$ $\quad 6+(8+7)=\square$ $\quad 8+(6+7)=\triangle$

$(6+7)+8=\square$ $\quad (7+6)+8=\triangle$ $\quad 8+(7+6)=\square$

Do you get the same sum in each case? Did we use both the commutative and the associative principles in these sentences?

2. Is "$20+5$" another name for 25? What is a similar name for 43? Then, we can work an addition example as follows.

$25+43=(20+5)+(40+3)$ renaming 25 and 43
$=(20+40)+(5+3)$ commutative and associative
$=60+?$ principles
$=?$

3. Use a similar method to show that $63+24=87$.

4. Use a similar method to show that $43+22+14=79$.

5. Do you see that you have been adding the ones and the tens separately? In Exercise **a** below, is the sum of the ones less than 10?

a.
$47=40+7$
$+28=20+8$
$\longrightarrow \quad 60+15$, or $\square$

b.
47
28
$?5$

Since 15 is $10+5$, can we think of $60+15$ as $60+(10+5)$? as $(60+10)+5$? What is the sum? In the short form (**b**), what digit should be written in place of the question mark?

Add down to find the sums. Can you check by adding upward? Why?

	a	b	c	d	e
6.	57	30	79	273	976
	97	38	45	780	43
	55	68	78	429	88
7.	3685	6000	8953	7626	278
	4997	307	939	7250	5650
	4085	995	39	5478	6208

1. If you are given two whole numbers and asked to find their sum, you add.

How do you find the missing addend in this sentence: $\square + 16 = 27$?

13	Addend
+ 12	Addend
25	Sum

Is this the related subtraction sentence: $27 - 16 = \square$?

2. To find a missing addend, you subtract. The answer in subtraction is called the *difference*. What is the difference between 27 and 16?

3. Find the missing numeral in each of the following.

a. $5 + N = 17;\ 17 - 5 = N$ **b.** $N + 13 = 29;\ 29 - 13 = N$

c. $16 + N = 39;\ 39 - 16 = N$ **d.** $N + 18 = 49;\ 49 - 18 = N$

For every addition sentence such as $N + 18 = 49$, there is a related subtraction sentence such as $49 - 18 = N$.

4. This sentence says that a certain number minus 32 is 12. Will you add, or will you subtract, to find the missing numeral? Do it.

$N - 32 = 12$

5. Study this sentence. Do you know that 53 must be the sum of 11 and the number represented by the $\square$? Find the missing numeral.

$53 - \square = 11$

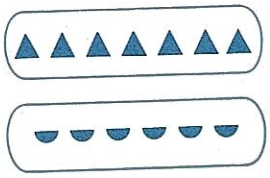
Find the missing numerals.

	a	b	c
6.	$N + 16 = 58$	$\square - 62 = 23$	$75 - \triangle = 51$
7.	$N - 102 = 203$	$726 + 810 = N$	$\square = 236 + 422$
8.	$\square + 380 = 695$	$\triangle - 412 = 115$	$208 + \square = 208$
9.	$790 - \triangle = 790$	$N - 735 = 111$	$N + 273 = 389$

10. Now study this pair of related sentences. Is there a whole number that you can add 8 to and get 1? Then does $1 - 8$ name a whole number? Does $2 - 5$ name a whole number? $4 - 9$? $6 - 15$?

$1 - 8 = N$ $N + 8 = 1$

1. Suppose you start with this set of 7 and join the set of 6 to it. How many members does the new set have?



$$7 + 6 = \square$$

Now suppose a set of 6 is removed from the set of 13. How many members will there be in the set that is left?

$$13 - 6 = \triangle$$

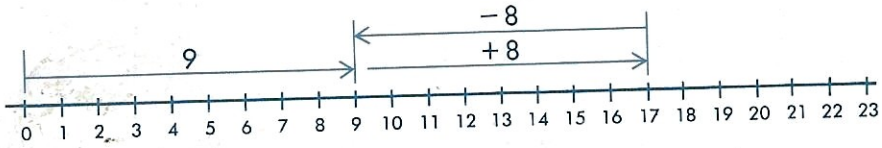
What is the missing numeral in this sentence?

$$(7 + 6) - 6 = N$$

Does 7 name the number you started with? Did subtracting 6 "undo" what was done by adding 6?

▶ Subtracting is the inverse of adding.

2. The number line also helps to illustrate what addition means and how subtracting undoes adding.



a. To show the sum of 9 and 8, start at the point labeled 0 and move 9 spaces to the right. To add 8, move 8 more spaces to the right. What is the numeral at the point where the second move ended? Then $9 + 8 = 17$.

b. You have just added 8 to 9; now take 8 from 17 by moving 8 spaces to the left. At what numeral did the last move end? What does $(9 + 8) - 8$ equal?

3. Find the missing numerals.

a. $(12 + 17) - 17 = \square$

b. $(54 - 25) + 25 = \triangle$

c. $(243 + 135) - 135 = \diamond$

d. $(679 - 220) + 220 = \square$

e. $(338 + 110) - \square = 338$

f. $(189 - 62) + \triangle = 189$

THINKING IT THROUGH

1. Study this addition example. What two-place numeral is the correct replacement for the □?

$$\begin{array}{rl} 54 & \text{is 5 tens, 4 ones} \\ +\,38 & \text{is 3 tens, 8 ones} \\ \longrightarrow & \text{8 tens, 12 ones, or } \square \end{array}$$

Below is the related subtraction example. Can you subtract 8 from 2? Then you must rename 92.

$$\begin{array}{rl} 92 & 92 \text{ is 9 tens, 2 ones which is 8 tens, ? ones} \\ -\,38 & -\,38 \text{ is 3 tens, 8 ones} \qquad\qquad 3 \text{ tens, 8 ones} \\ & \longrightarrow \qquad\qquad ? \text{ tens, ? ones, or } \triangle \end{array}$$

What is the correct replacement for the △?

2. Find the sum and the difference. Show the renaming.

a. $\begin{array}{r} 442 \\ +\,329 \\ \hline \end{array}$ **b.** $\begin{array}{r} 871 \\ -\,329 \\ \hline \end{array}$

3. Can you do the renaming mentally? Copy the examples and follow the thinking as you work them.

	Example	Thinking		Example	Thinking
a.	529 +572	9+2=? 1+2+7=? 1+5+5=?	**b.**	1101 −572	11−2=? 9−7=? 10−5=?

4. Study this example. How many cents is \$8.55? How many cents is \$3.68? Subtract as if you are subtracting 368 from 855.

$$\begin{array}{r} \$8.55 \\ -\,3.68 \\ \hline \end{array}$$

Put a dollar sign and a point in the answer to show that it means money. Check by adding.

Find the sums or differences and check.

5.a. $\begin{array}{r} 636 \\ -\,257 \\ \hline \end{array}$ **b.** $\begin{array}{r} 8720 \\ -\,6892 \\ \hline \end{array}$ **c.** $\begin{array}{r} 4752 \\ -\,3523 \\ \hline \end{array}$ **d.** $\begin{array}{r} 808 \\ -\,253 \\ \hline \end{array}$ **e.** $\begin{array}{r} 3581 \\ -\,2611 \\ \hline \end{array}$

6.a. $\begin{array}{r} 637 \\ 59 \\ 704 \\ \hline \end{array}$ **b.** $\begin{array}{r} 6545 \\ 6636 \\ 25 \\ \hline \end{array}$ **c.** $\begin{array}{r} 8000 \\ 3768 \\ 4469 \\ \hline \end{array}$ **d.** $\begin{array}{r} 5269 \\ 874 \\ 7210 \\ \hline \end{array}$ **e.** $\begin{array}{r} 3620 \\ 6100 \\ 2005 \\ \hline \end{array}$

Copy, add, and check.

	a	b	c	d	e
1.	547 886 957	718 689 979	834 266 902	963 476 561	4426 3901 7528
2.	\$3.65 2.80 5.25	\$3.00 .78 1.20	\$3.75 .29 .05	\$1.84 7.40 .07	\$86.75 17.33 .12
3.	8,578 269 28,803 57,950	83,507 54,636 79,070 62,853	64,978 800 759 28,074	91,576 2,673 8,359 45,030	80,040 31,798 8,500 694

Copy, subtract, and check.

4.	900 548	614 367	7000 2484	9098 689	3005 2346
5.	\$84.22 40.37	\$50.45 28.96	\$90.00 64.13	\$59.30 50.41	\$123.65 89.75
6.	59,146 43,899	72,844 6,895	53,969 39,170	38,275 15,287	68,444 24,468

Using what you know about addition and subtraction, find the missing numerals.

	a	b	c
7.	$\square - 15 = 46$	$\triangle + 38 = 71$	$42 - \diamond = 26$
8.	$212 + \square = 300$	$700 - \square = 275$	$231 + \square = 802$
9.	$\square + 115 = 204$	$\triangle - 38 = 162$	$\square - 842 = 524$
10.	$\triangle + 355 = 640$	$\square - 402 = 398$	$\square = 726 + 382$

Tell whether each sentence is true, or false.

	a	b
11.	$5_{six} + 10_{six} = 10_{six} + 5_{six}$	$202_{six} + 0_{six} = 203_{six}$
12.	$4_{six} + 2_{six} + 11_{six} = 11_{six} + 2_{six} + 5_{six}$	$100_{six} - 1_{six} < 100_{six}$

22

PROBLEM SOLVING

In solving problems, number sentences are often helpful to show what numbers are given and what we want to find.

1. There were 27 children in the sixth-grade class, but one day only 19 were present. How many were absent?

A number sentence for this problem is: "Twenty-seven minus nineteen equals what number?" Write the sentence, using digits and N instead of words. Then find the answer to the question in the problem.

Read each problem below. For the first five problems, write number sentences to help you solve them, but solve Problem **7** without writing a number sentence.

2. Students were selected from the three sixth-grade classes to sing in the chorus at the school concert. There were 19 children from Miss Smith's room, 17 from Mrs. Moon's, and 13 from Miss Nelson's. How many students were selected?

3. Miss Smith's class had $20 to spend for parties during the school year. After buying refreshments for their first party, they had $14.05 left. How much did they spend?

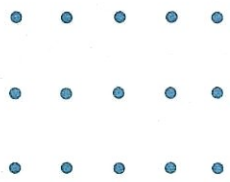
4. The principal's office announced that 83 boys and girls were taking instrumental music this year. This is 17 more than last year. How many took instrumental music last year?

5. One day the girls gathered leaves for a science project. Ann found 23 different leaves, Susan found 31, and Marie found 29. How many leaves did the girls gather?

6. During one week, just after the new models appeared, the production of three makes of cars was: Chevrolet, 32,264; Ford, 28,788; Plymouth, 5832. How many cars of these three makes were produced that week?

7. Each day Lynn's father gave her all of his nickels and pennies. Monday she got 3 nickels and 3 pennies; Tuesday, 17¢; Wednesday, 6 nickels and 9 pennies; Thursday, nothing. On Friday her father gave her 3 nickels and 7 pennies. How much money did Lynn get in all?

1. Study this array. How many rows are there? How many dots are there in each row? What is 3×5? Is 15 the number of dots in the array? Are 3 and 5 factors of 15?



2. Now find the number of dots in another way. Think of the columns first. How many columns are there? How many dots are there in each column? What is 5×3?

Are 3×5 and 5×3 names for the same number?

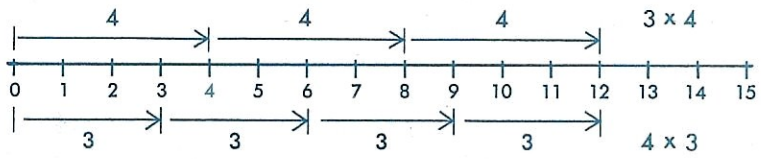
3. Make an array that has 4 rows and 6 columns.

a. What is 4×6? 6×4?

b. Is this sentence true, or is it false: $4 \times 6 = 6 \times 4$? Does the order in which factors are used change the product?

▶ If $\square$ and $\triangle$ represent any whole numbers, then $\square \times \triangle = \triangle \times \square$. This sentence states the **commutative principle of multiplication.**

4. We can also use a number line to help us think about multiplication.



Does the number-line drawing show that multiplication of whole numbers can be thought of as repeated addition? What one numeral should replace each $\square$ in sentence **b**?

a. $3 \times 4 = 4 + 4 + 4$ **b.** $4 \times 3 = \square + \square + \square + \square$

5. Make number line drawings to show 4×5; 5×4.

REVIEWING MULTIPLICATION—continued

6. Find the missing numeral in each of the following.

a. $4 \times 1 = N$ **b.** $1 \times \square = 13$ **c.** $8 \times N = 8$

Does the product of a whole number and 1 equal that number? Is the product of 1 and any whole number equal to that number?

▶ The **identity element for multiplication** of whole numbers is 1, since $N \times 1 = N$ and $1 \times N = N$.

In each of the following, find the correct replacement for N.

7.a. $(3 \times 2) \times 2 = N$ **b.** $3 \times (2 \times 2) = N$

8.a. $(4 \times 2) \times 5 = N$ **b.** $4 \times (2 \times 5) = N$

Do you get the same product in **7a** and **7b**? in **8a** and **8b**?

▶ If $\square$, $\triangle$, and $\diamond$ represent any whole numbers, then $\square \times (\triangle \times \diamond) = (\square \times \triangle) \times \diamond$. This sentence states the **associative principle of multiplication.**

9. What is 3×0? 0×8? 9×0? If N represents any whole number, $N \times 0 = 0$ and $0 \times N = 0$.

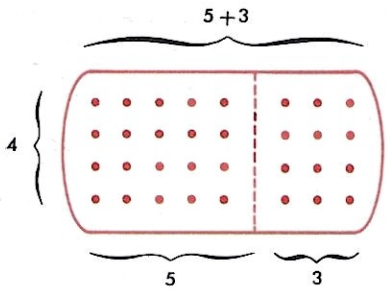
Use your understanding of multiplication to find the missing numerals.

	a	b
10.	$3 \times 9 = \square \times 3$	$71 \times 6 = 6 \times \triangle$
11.	$212 \times N = 0$	$212 \times N = 212$
12.	$2 \times 90 = 2 \times (9 \times \square)$	$2 \times 90 = (2 \times 9) \times \square$
13.	$3 \times 70 = 3 \times (7 \times \square)$	$3 \times 70 = (3 \times \triangle) \times 10$
14.	$5 \times 300 = 5 \times (\square \times 100)$	$5 \times 300 = (5 \times \square) \times 100$
15.	$6 \times 200 = 6 \times (2 \times \triangle)$	$6 \times 200 = (6 \times \diamond) \times 100$

25 THE DISTRIBUTIVE PRINCIPLE

1. Study this array. Does $5+3$ name the number of dots in each row? Then does $4\times(5+3)$ tell the number of dots in this array?

The array has been partitioned into two subsets. What is the missing numeral in this sentence?



$$4\times(5+3)=(4\times 5)+(4\times\triangle)$$

2. Draw a partitioned array to show each of the following. Then multiply to show that the sentence is true.

a. $3\times(8+2)=(3\times 8)+(3\times 2)$

b. $4\times(7+2)=(4\times 7)+(4\times 2)$

▶ If N, □, and △ represent any whole numbers, then
$N\times(\square+\triangle)=(N\times\square)+(N\times\triangle)$.

This sentence states the **distributive principle** of multiplication over addition.

3. Is "$20+5$" another name for 25? Do you think $4\times 25=(4\times 20)+(4\times 5)$? Try it and see.

4. Rename 25 as the sum of any three numbers. Now multiply each of the numbers by 4 and add the products. What answer do you get? Does it seem that in multiplying you can rename a number as the sum of several numbers, multiply each of these, and add the products?

5. What is the missing numeral in **a**? in **b**?

a. $4\times 32=(4\times\square)+(4\times 2)$

b. $5\times 69=(5\times 60)+(5\times\triangle)$

Use the distributive principle to find the products.

6.a. 4×32 **b.** 5×69 **c.** 3×23

7.a. 6×45 **b.** 9×24 **c.** 4×82

MULTIPLICATION WITH RENAMING

1. Could you find the product of 7 and 224 by finding 7×200, 7×20, and 7×4 and then adding these products? What principle connecting multiplication and addition are you using?

Copy and complete **a** and **b**. In the vertical form, which do you multiply first—the ones, or the tens?

a

$$7 \times 224 = (7 \times \square) + (7 \times \triangle) + (7 \times \diamond)$$
$$= \quad ? \quad + \quad ? \quad + \quad ?$$
$$= \quad N$$

b

$$\begin{array}{r} 224 \\ \times\ 7 \\ \hline 28 \quad (7 \times \diamond) \\ 140 \quad (7 \times \triangle) \\ 1400 \quad (7 \times \square) \end{array}$$

2. Use the method of Exercise **1** to find the products.

a. $5 \times 69 = N$ **b.** $8 \times 74 = N$ **c.** $4 \times 759 = N$

d. $7 \times 468 = \square$ **e.** $6 \times 356 = \triangle$ **f.** $7 \times 937 = \diamond$

3. You know a short form for working multiplication examples. What is 7×4? You write the 8 and remember the 2 tens. Explain the 6 in tens' place. Copy and complete the example.

Short Form

$$\begin{array}{r} 224 \\ \times\ 7 \\ \hline ??68 \end{array}$$

4. Work this example, using the short form. What is 5×7? Write the 5 and remember the 3 tens. What is 5×0? Add the 3 tens you are remembering to the 0 tens. What do you write in tens' place? What is 5×8? What do you write in hundreds' place? in thousands' place?

$$\begin{array}{r} 807 \\ \times\ 5 \\ \hline ???? \end{array}$$

Find the products, using the short form.

	a	**b**	**c**	**d**	**e**
5.	219 × 5	802 × 4	742 × 3	503 × 5	418 × 4
6.	251 × 6	400 × 9	609 × 3	618 × 4	967 × 2

7.a. $8 \times 118 = N$ **b.** $3 \times 899 = N$ **c.** $9 \times 1002 = N$

27 USING THE COMMUTATIVE AND ASSOCIATIVE PRINCIPLES

1. What does 10×3 equal? 10×5? 10×9? 10×13? 10×27? Do you see a pattern you can use when you multiply a number by 10? Then what is 10×31? 10×35? 10×40?

2. What does 100×4 equal? 100×6? 100×9? 100×12? 100×23? Do you see a pattern you can use when you multiply a number by 100? Then tell what 100×56 equals; 100×84; 100×95; 100×107.

3. Now use this exercise to find what 20×40 equals.

$$\begin{aligned} 20 \times 40 &= (2 \times 10) \times (4 \times 10) \\ &= (2 \times 4) \times (10 \times 10) \\ &= \square \times 100 \\ &= N \end{aligned}$$

What principles of multiplication allow you to rearrange factors and group them differently?

4. Study **a** below. To find what 30×42 equals, can you use the commutative and associative principles and concentrate on 3×42?

a.
$$\begin{aligned} 30 \times 42 &= (3 \times 10) \times 42 \\ &= (3 \times 42) \times 10 \\ &= N \end{aligned}$$

b.
$$\begin{aligned} 400 \times 132 &= (4 \times 100) \times 132 \\ &= (4 \times 132) \times 100 \\ &= \square \end{aligned}$$

Now use Exercise **b** to find what 400×132 equals.

Find the products.

	a	b	c	d
5.	30×50	40×70	60×90	80×80
6.	20×26	40×74	30×45	60×27
7.	50×99	70×86	20×95	90×88
8.	200×72	400×312	500×66	300×83
9.	100×100	200×300	400×400	600×700
10.	600×738	500×139	700×2645	300×6523

28 ANOTHER LOOK AT THE DISTRIBUTIVE PRINCIPLE

1. Jim showed his younger brother how to find what 8×9 is. He renamed the second factor as $5 + 4$, and wrote the sentence below.

$$8 \times (5 + 4) = (8 \times 5) + (8 \times 4)$$

What principle was Jim using?

Instead of renaming the second factor in 8×9, could he have renamed the first factor, 8, and used this exercise? Do the work yourself and see.

$$\begin{aligned}(6 + 2) \times 9 &= (6 \times 9) + (2 \times 9) \\ &= \square + \triangle \\ &= N\end{aligned}$$

2. To find the product of 34×72, we use the commutative, associative, and distributive principles. Is "$30 + 4$" another name for 34? Is it true that $30 + 4 = 4 + 30$? Why?

a

$$\begin{aligned}34 \times 72 &= (30 + 4) \times 72 \\ &= (4 + 30) \times 72 \\ &= (4 \times 72) + (30 \times 72) \\ &= \square + \triangle \\ &= N\end{aligned}$$

b

$$\begin{array}{r} 72 \\ \times\ 34 \\ \hline 288 \longleftarrow (4 \times 72) \\ 2160 \longleftarrow (30 \times 72) \end{array}$$

3. Is it true that $34 \times 72 = 72 \times 34$? Why? Then you can check your work by multiplying 34 by 72. Copy, and complete the check.

$$\begin{array}{r} 34 \\ \times\ 72 \\ \hline \end{array}$$

Copy, multiply, and check.

	a	b	c	d	e	f	g
4.	43	96	58	46	62	75	92
	$\times$ 95	$\times$ 57	$\times$ 29	$\times$ 53	$\times$ 86	$\times$ 49	$\times$ 38
5.	73	94	86	57	98	84	76
	$\times$ 67	$\times$ 39	$\times$ 62	$\times$ 84	$\times$ 78	$\times$ 53	$\times$ 67

6.a. 38×48 **b.** 74×58 **c.** 46×68 **d.** 59×65

1. In Example **a**, the method shown is the method you have been using. In **b**, the form has been shortened. Is the product of 8 and 74 shown in the same way in both **a** and **b**?

a.		b.	
	74		74
	28		28
	592		592
	1480		148
	2072		2072

Now compare the way the product of 20 and 74 is shown in **a** and **b**. How do they differ? When you multiply by 10, 20, 30, and so on, does the product always have a 0 in ones' place? In the short form, we omit the 0. What does the "148" mean?

2. Copy and multiply. Check by multiplying again.

a.	**b.**	**c.**	**d.**	**e.**	**f.**
25	84	73	64	59	85
78	47	14	37	63	18

3. A box of typing paper costs $3.85. How much will 12 boxes of this typing paper cost? Will you add, subtract, or multiply to find the answer?

How many cents is $3.85? Think of the $3.85 as 385 and multiply just as you would with any numbers.

Copy the example and complete it. Remember to place a dollar sign and a point in the answer. What is the cost of the 12 boxes of typing paper?

$3.85
12
770
???

Copy and multiply. Place a dollar sign and a point in your answer whenever necessary.

	a	b	c	d	e
4.	$2.87	$4.78	$14.80	$31.74	$64.19
	9	8	6	3	7
5.	$8.36	307	580	809	$1.49
	14	87	59	47	29

1. Without writing an example, do you know that this sentence is true? How? **$49 \times 68 < 3500$**

Think of 49 as about 50 and 68 as about 70. Is it easy now to see that the sentence is true? Why?

When you think of 49 as about 50 (or 68 as about 70) you are **rounding** the number to the nearest ten.

2. Is 34 closer to 30 or to 40? What is 34 rounded to the nearest ten? What is 36 rounded to the nearest ten?

Is a number such as 35 midway between 30 and 40? By agreement, such "midway numbers" are rounded upward. What is 35, rounded to the nearest ten? What is 65 rounded to the nearest ten?

3. Round each of the following to the nearest ten.

a. 74	**b.** 75	**c.** 175	**d.** 163	**e.** 148
f. 85	**g.** 214	**h.** 215	**i.** 321	**j.** 325

4. The part of the number line below shows certain numerals corresponding to points on the line. The space between 300 and 400 is divided into one hundred spaces.

Is 343 closer to 300 or to 400? Then what is 343 rounded to the nearest hundred? Is 361 closer to 300 or to 400? What is 361 rounded to the nearest hundred?

Is the number 350 midway between 300 and 400? Then we round 350 to 400.

5. Round each of the following to the nearest hundred.

a. 306	**b.** 453	**c.** 450	**d.** 742	**e.** 299
f. 856	**g.** 849	**h.** 1838	**i.** 1455	**j.** 2850

6. Without writing any examples, tell whether each sentence is true or false.

a. $78 \times 78 > 6400$ **b.** $61 \times 43 < 2400$

c. $197 \times 878 < 180{,}000$ **d.** $250 \times 350 < 120{,}000$

1.a. What is 3×117? 50×117? 200×117?

b. What is 2×629? 40×629? 300×629?

2. As you answer these questions about the examples at the right, look back to **1a** and **1b** as a guide.

In **a**, what does the "585" mean? What does the "234" mean?

In **b**, what does the "2516" mean? the "1887"?

a.

$$\begin{array}{r} 117 \\ 253 \\ \hline 351 \\ 585 \\ 234 \\ \hline 29{,}601 \end{array}$$

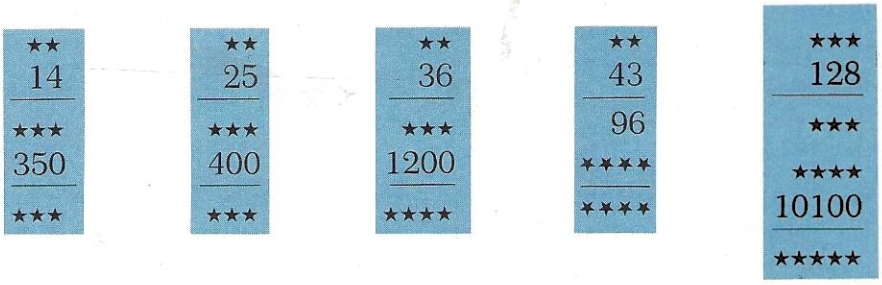
b.

$$\begin{array}{r} 629 \\ 342 \\ \hline 1\,258 \\ 2516 \\ 1887 \\ \hline 215{,}118 \end{array}$$

Copy, and multiply.

	a	b	c	d	e
3.	468 316	325 248	648 725	372 165	586 957
4.	573 264	792 503	854 408	913 767	508 946

5. Each box below has a "mystery" multiplication example. Can you discover the correct numerals from the clues that are given?



6. What numeral will make this sentence true? Remember, when the same frame is used more than once in a sentence, it stands for the same number. $\square \times \square = 2500$

7. Find the missing numerals.

a. $\triangle \times \triangle = 10{,}000$ **b.** $\square \times \square = 900$ **c.** $\diamond \times \diamond = 80^2$

1. Copy these sentences and write the missing numerals.

a. $N \times 7 = 28;\ 28 \div 7 = N$ **b.** $N \times 4 = 28;\ 28 \div 4 = N$

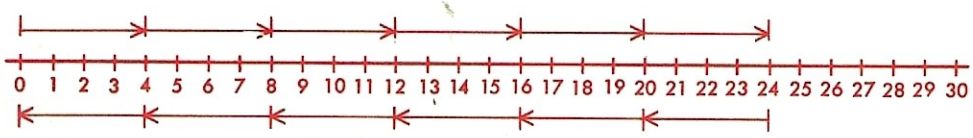
c. $N \times 9 = 72;\ 72 \div 9 = N$ **d.** $N \times 8 = 72;\ 72 \div 8 = N$

For any multiplication sentence, such as $7 \times 6 = 42$, there is a related division sentence: $42 \div 6 = 7$. Dividing is the **inverse** of multiplying.

2. In each multiplication sentence below, there is a missing factor. Write a related division sentence to find the missing factor.

a. $N \times 3 = 27$ **b.** $7 \times N = 140$ **c.** $8 \times N = 240$

3. The arrows above the number line illustrate the number sentence $6 \times 4 = \square$. What is the missing numeral?

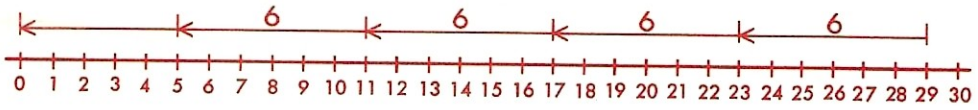


The arrows below the number line illustrate the sentence $24 \div 4 = N$. Where do you start? In which direction do you move? How many moves of 4 do you make to get to 0? What is the correct replacement for N?

4. You know that multiplication of whole numbers can be thought of as repeated addition. Does the number-line drawing show that division can be thought of as repeated subtraction?

5. Here are the names given to numbers in a division relationship. What is the missing numeral in this sentence?

Make a number-line drawing to show that $20 \div 5$ is 4.



1. This drawing illustrates what happens when we divide 29 by 6. Starting at 29, how many times can you make moves of 6 toward 0? Can you get to 0 by using moves of 6?

After you make four moves of 6 spaces each, do 5 spaces remain?

2. If you subtract 4 sixes from 29, do you get 5? When you divide 29 by 6, the quotient is 4, and the 5 that is left is the **remainder**.

In division of whole numbers, we can think of subtracting the divisor from the dividend as many times as possible. Here is a sentence we can use for Exercise **1**.

$$29 - (N \times 6) = 5$$

Does N represent the quotient? What is the dividend? the divisor? the remainder? What is the correct replacement for N?

Find the quotient and remainder for each of the pairs of numbers named in Exercises **3–7**.

	Dividend	Divisor	Quotient	Remainder
3.	33	5	?	?
4.	68	9	?	?
5.	82	8	?	?
6.	81	8	?	?
7.	80	8	?	?

8. Here is a sentence for the division you did in Exercise **3**.

$$33 - (6 \times 5) = 3$$

Write similar sentences for Exercises **4–7**.

9. In Exercise **7**, did you find that the quotient was 10 with a remainder of 0? When the remainder is 0, are the divisor and the quotient *factors* of the dividend? What is 10×8?

A DISCOVERY ABOUT DIVISION

1. What is the quotient when 86 is divided by 2? Then what numeral will make this sentence true? **$86 \div 2 = N$**

2. Is $80 + 6$ another name for 86? What is $80 \div 2$? What is $6 \div 2$? What is $40 + 3$? Does the exercise below show a way of finding the quotient for $86 \div 2$?

$$\begin{aligned} 86 \div 2 &= (80 + 6) \div 2 \\ &= (80 \div 2) + (6 \div 2) \\ &= \quad 40 \quad + \quad 3, \text{ or } N \end{aligned}$$

3. Look at this division sentence: $763 \div 7 = N$. Is $700 + 63$ a name for 763? Do you think we can find what $763 \div 7$ is by dividing the 700 by 7 and the 63 by 7, and then adding these quotients? Use the exercise below to see.

$$\begin{aligned} 763 \div 7 &= (700 + 63) \div 7 \\ &= (700 \div 7) + (63 \div 7) \\ &= \quad ? \quad + \quad ?, \text{ or } N \end{aligned}$$

What is the quotient? Check by multiplying: $109 \times 7 = \square$.

4. To find what $642 \div 3$ is, we can name the dividend as the sum of numbers that can be easily divided by 3. Use the exercise below to do this.

$$\begin{aligned} 642 \div 3 &= (600 + 30 + \square) \div 3 \\ &= (600 \div 3) + (30 \div 3) + (\square \div 3) \\ &= \quad ? \quad + \quad ? \quad + \quad ?, \text{ or } N \end{aligned}$$

Check by finding the product of the quotient and the divisor.

Using the method of Exercises **3–4**, find the quotients.

	a	b	c	d
5.	$639 \div 3$	$168 \div 2$	$264 \div 2$	$175 \div 5$
6.	$990 \div 9$	$808 \div 8$	$486 \div 6$	$672 \div 3$
7.	$7\overline{)4907}$	$8\overline{)648}$	$6\overline{)4818}$	$9\overline{)5409}$
8.	$9\overline{)4518}$	$4\overline{)1204}$	$5\overline{)2215}$	$8\overline{)3272}$

1. At the right is a form for division that you have used in earlier grades.

a. Are there as many as 20 twelves in 420? as many as 30 twelves? as many as 40 twelves?

```
12)420
360 30
 60
 60  5
```

Did you find that there are as many as 30 twelves but not as many as 40 twelves? We write 30 at the side. What is 30×12? We subtract 360 from 420 to find what there is left to divide.

b. How many twelves are there in 60? We write the 5 at the side. What is 5×12?

What is the quotient?

2. Now work the example in your usual way. Is 3 the first digit you write in the quotient? Does this "3" mean 3, or does it mean 30?

```
  3
12)420
 36
```

Since the 3 means 30, what does the "36" mean?

3. Here are two partly completed division examples.

a.
```
   4
13)546
 52
```

b.
```
   2
19)399
 38
```

In **a**, a 4 has been written above tens' place in the dividend. Does this "4" mean 4, or does it mean 40? What is 40×13? What does the "52" mean?

In **b**, a 2 has been written above tens' place in the dividend. What does the "2" mean? What does the "38" mean?

Copy and work examples **a** and **b** above.

Divide and check.

	a	b	c	d
4.	$21\overline{)651}$	$81\overline{)972}$	$45\overline{)585}$	$23\overline{)966}$
5.	$32\overline{)768}$	$52\overline{)988}$	$63\overline{)882}$	$27\overline{)621}$
6.	$39\overline{)819}$	$58\overline{)2610}$	$72\overline{)936}$	$41\overline{)2460}$

36 MORE DIVISION WITH REMAINDERS

1. Which of the following sentences are true? Which is false?

a. $40 \times 15 < 790$ **b.** $50 \times 15 < 790$ **c.** $60 \times 15 < 790$

Then must the quotient for the example $15\overline{)790}$ be a number greater than 50 but less than 60?

2. At the right, the division example $15\overline{)790}$ has been worked in the long form.

a. What is 50×15?

b. What is $790 - 750$?

c. How many fifteens are there in 40? What is 2×15? What is $40 - 30$?

What do we call the 10 we have left? What is the quotient for the example? We write the answer as **52 R10**.

?? R?	
15)790	
750	50
40	
30	2
10	

3. To check this division example, can you just multiply: $52 \times 15 = \diamond$? What must you do next?

4. Work this example in your usual way.

a. What is the quotient? the remainder? Check your answer.

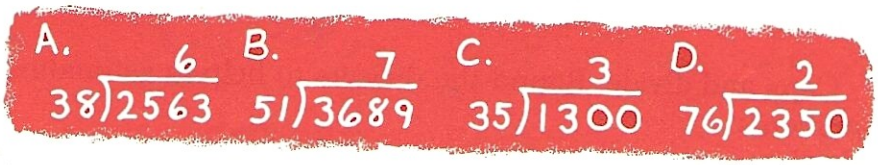
$20\overline{)783}$

b. Suppose we use D to represent the dividend, d to represent the divisor, Q to represent the quotient, and R to represent the remainder. Does this sentence tell about the relationship in division of whole numbers?

$$D = (Q \times d) + R$$

Divide. Check all answers using a sentence like the one shown in color.

	a	b	c	d
5.	$30\overline{)889}$	$40\overline{)1855}$	$60\overline{)1392}$	$50\overline{)1771}$
6.	$15\overline{)976}$	$12\overline{)493}$	$48\overline{)987}$	$25\overline{)862}$
7.	$21\overline{)1281}$	$32\overline{)1664}$	$44\overline{)1369}$	$80\overline{)1258}$
8.	$90\overline{)1894}$	$52\overline{)1700}$	$39\overline{)1209}$	$70\overline{)2240}$



1. This illustration shows chalkboard work done by four pupils. Which is easier to estimate—the number of thirty-eights in 2563 or the number of forties in 2563? In example **A**, the pupil rounded the divisor to the nearest ten; he thought of 38 as about 40. What does the quotient digit "6" mean? Do you think it is the correct estimate: $6 \times 38 = \square$? Copy the division example and complete it. What does the 228 in the completed example mean?

2. In **B**, if we round the divisor to the nearest ten, what number do we use to estimate? What does the quotient digit "7" mean? Do you think this is the correct estimate? How can you find out whether it is? Copy and complete the example.

3. In **C**, would you round 35 to 30, or to 40? Do you think the quotient digit "3" is correct? Work the example to find out.

4. Look at **D**. What is 76 rounded to the nearest ten? How was the quotient digit "2" obtained? When you multiply 76 by 2, what do you get? When you subtract "152" from "235," what do you get? Is there something wrong?

Is the following sentence true: $30 \times 76 < 2350$? Then should the quotient digit be 3?

Do you see that when we use a rounded divisor to estimate, we do not always get the correct quotient digit? Work the whole example yourself.

Divide. Round the divisors to help in estimating.

	a	b	c	d
5.	79)5228	24)1996	23)1162	37)2621
6.	32)2400	55)2617	31)2586	28)2566
7.	47)3258	82)6153	85)7400	55)4085

THINKING IT THROUGH

Copy and divide. Round the divisors to help in estimating quotients.

	a	b	c	d
1.	$48\overline{)1313}$	$35\overline{)2676}$	$59\overline{)2617}$	$27\overline{)1257}$
2.	$87\overline{)6927}$	$45\overline{)2925}$	$36\overline{)2026}$	$86\overline{)2431}$

In Exercises **1** and **2**, did you round each divisor upward? Was any quotient estimate too large? Were some too small?

Copy and divide. Round the divisors in estimating.

3.	$81\overline{)6561}$	$64\overline{)5135}$	$43\overline{)2432}$	$52\overline{)3886}$
4.	$42\overline{)1624}$	$24\overline{)1392}$	$93\overline{)6666}$	$32\overline{)1839}$

In Exercises **3** and **4**, did you round each divisor downward? Was any quotient estimate too small? Were some too large?

5. You have been rounding divisors to estimate quotients. Have you found a system of your own that makes estimating easier? If so, tell about it, using **a** or **b**.

a. $48\overline{)1566}$ **b.** $24\overline{)1256}$

For **a**, one system that some people use is to think of 48 as 5 tens, and 1566 as "150 tens and 66." What is 150 tens ÷ 5 tens? Work the example.

For **b**, some people do not round the divisor. They think: "There are about 5 twenty-fours in 125; then about how many twenty-fours are there in 1256?" Work the example.

Copy, divide, and check.

	a	b	c	d
6.	$43\overline{)2434}$	$28\overline{)1802}$	$45\overline{)2600}$	$43\overline{)2150}$
7.	$54\overline{)2516}$	$59\overline{)3106}$	$76\overline{)4925}$	$95\overline{)2850}$
8.	$37\overline{)2135}$	$85\overline{)3210}$	$67\overline{)4260}$	$54\overline{)3264}$
9.	$93\overline{)3772}$	$74\overline{)2860}$	$69\overline{)4395}$	$76\overline{)5350}$

For Exercises **1–3**, choose each answer from this set of numbers: {10, 20, 30, 40, 50, 60, 70, 80, 90}.

Rewrite each sentence, using the numeral for the greatest number that will make the sentence true.

	a	b
1.	$N \times 53 < 2000$	$N \times 91 < 8000$
2.	$N \times 36 < 1250$	$N \times 48 < 2565$
3.	$N \times 83 < 4008$	$N \times 76 < 6055$

4. What is the missing numeral in this sentence: $30 \times N = 600$?

5. In your work you have used sentences that have the signs $=$, $<$, and $>$. Sentence **a** uses the $=$ sign and is called an **equation**. The signs $<$ and $>$ are *inequality signs*, and sentences such as **b** and **c** are called **inequalities**. What kind of sentence is $\square \div 40 = 80$? What numeral will make the sentence true?

a. $30 \times N = 600$

b. $N \times 53 < 2000$

c. $1000_{six} > 100$

6. Solve each of the following equations. That is, find the name of the number that will make the sentence true.

$N = (40 \times 72) + 3$ $\qquad$ $N = (33 \times 64) + 7$

What is the quotient and remainder for this example: $72\overline{)2883}$? for this example: $64\overline{)2119}$?

For Exercises **7–9**, choose each answer from this set of numbers: {100, 200, 300, 400, 500, 600, 700, 800, 900}.

Rewrite each inequality, using the numeral for the greatest number that will make the sentence true.

	a	b
7.	$N \times 62 < 20{,}000$	$N \times 83 < 18{,}500$
8.	$N \times 91 < 10{,}268$	$N \times 32 < 26{,}500$
9.	$N \times 77 < 40{,}750$	$N \times 55 < 24{,}326$

TESTING YOUR UNDERSTANDING

1. Are there more than 100 forty-eights in 10,576? Which of the following sentences are true? Which are false?

48)10,576

a. $100 \times 48 < 10{,}576$ **b.** $200 \times 48 < 10{,}576$

c. $300 \times 48 < 10{,}576$ **d.** $400 \times 48 < 10{,}576$

What is the first quotient digit for the division example? Where will you write it—above the 0, the 5, or the 7?

Copy and complete the example.

2. A troop of 31 Girl Scouts collected a total of \$475.25 from the sale of cookies. What was the average amount of sales per Girl Scout in this troop?

31)\$475.25

Can you think of \$475.25 as 47,525 cents? Is this number named by 47,000 + 525? Estimate the number of thirty-ones in 47,000. Are there as many as 10,000 thirty-ones? as many as 1000 thirty-ones? Where will you write the first quotient digit? Work the example carefully. Remember, your answer to the problem will need a dollar sign and a point.

3. Are there as many as 1000 forty-threes in 21,603? Then think of hundreds of forty-threes.

43)21,603

Look at this inequality. Will the sentence be true if we replace N by 200? by 300? by 400? by 500? by 600? What is the greatest number of hundreds that will make the sentence true?

$N \times 43 < 21{,}603$

Copy and work the division example.

Find the greatest number of thousands or of hundreds that will make each sentence true. Then work the example.

4.a. $N \times 8 < 12{,}652$ **b.** $8\overline{)12{,}652}$

5.a. $N \times 11 < 57{,}629$ **b.** $11\overline{)\$576.29}$

6.a. $N \times 39 < 28{,}432$ **b.** $39\overline{)28{,}432}$

7.a. $N \times 52 < 32{,}968$ **b.** $52\overline{)\$329.68}$

Divide and check.

	a	b	c	d
1.	$8\overline{)5658}$	$9\overline{)9604}$	$5\overline{)5810}$	$7\overline{)1656}$
2.	$18\overline{)3952}$	$26\overline{)7775}$	$24\overline{)9028}$	$49\overline{)50{,}263}$
3.	$37\overline{)22{,}659}$	$91\overline{)8687}$	$19\overline{)6435}$	$15\overline{)12{,}025}$
4.	$34\overline{)22{,}415}$	$16\overline{)33{,}876}$	$45\overline{)9438}$	$86\overline{)6355}$
5.	$72\overline{)53{,}024}$	$81\overline{)90{,}000}$	$17\overline{)6555}$	$93\overline{)85{,}899}$
6.	$25\overline{)54{,}301}$	$35\overline{)70{,}009}$	$61\overline{)50{,}000}$	$23\overline{)25{,}589}$
7.	$32\overline{)18{,}500}$	$74\overline{)85{,}699}$	$58\overline{)40{,}000}$	$39\overline{)20{,}001}$

8. Sentence **a** says that 150 divided by a certain number equals 5. Does this mean that 5 times that number is 150? Then are you looking for a missing factor in **b**? Find the missing numeral.

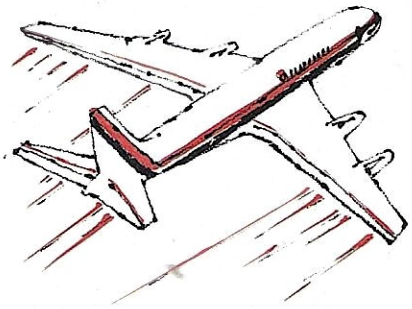
a. $150 \div \triangle = 5$

b. $5 \times \triangle = 150$

9. The sentence $\square \div 5 = 40$ says that a certain number divided by 5 equals 40. Write the related multiplication sentence. What is the missing numeral?

Using what you know about multiplication and division, solve these equations.

	a	b	c
10.	$9 \times \square = 180$	$\triangle \div 3 = 50$	$360 \div \triangle = 4$
11.	$\square \times 5 = 250$	$\diamond \times 3 = 636$	$N \div 4 = 215$
12.	$486 \div 3 = N$	$\square = 12 \times 18$	$7 \times \triangle = 2184$
13.	$N \div 7 = 96$	$252 \div \square = 6$	$382 \div N = 382$
14.	$\triangle \times 237 = 0$	$732 \div \square = 12$	$\triangle \div 15 = 15$
15.	$N \times 46 = 966$	$\square = 2520 \div 60$	$38 \times \square = 2280$



1. A jet transport made a trip of 6600 miles. The trip took 12 hours. On the average, how many miles did the jet travel in an hour?

Suppose you let N represent the average number of miles the jet traveled in an hour. Does this equation tell about the problem?

$$12 \times N = 6600$$

Does this equation also tell about the problem?

$$6600 \div 12 = N$$

Find the answer to the question asked in the problem.

For each of the following, write an equation that tells about the problem. Then solve the problem.

2. If a plane travels at an average speed of 575 miles an hour, how far will it travel in 15 hours?

3. The distance between Jim's house and the library is 1400 yards. If Jim walks at an average speed of 40 yards per minute, how many minutes will it take him to walk from his house to the library?

4. Suppose Jim wants to make the trip from the house to the library in 25 minutes. How many yards must he walk per minute?

5. A certain satellite had an orbit of about 27,000 miles. It took the satellite 95 minutes for an orbit. On the average, how many miles did the satellite travel in a minute?

6. The United States satellite *Syncom* 3 relayed pictures of the Tokyo Olympic games. It stayed at a fixed point above the earth. Its height ranged from 21,690 miles to 22,509 miles above the surface of the earth. What is the difference between these heights?

1. Is 3 a factor of 15? When you divide 15 by 3, do you get 0 as the remainder? $3 \times 5 = 15$ $3\overline{)15}$

Is 5 a factor of 15?

When you divide 15 by 6, is the remainder 0? Then 6 *is not* a factor of 15.

2. Tell whether each of the following is a factor of 8.

a. 1 **b.** 2 **c.** 3 **d.** 4 **e.** 5 **f.** 6 **g.** 7 **h.** 8

Does {1, 2, 4, 8} contain all the factors of 8?

3. Which of the following is the set of all factors of 12?

a. {1, 2, 3, 4, 6, 8, 12} **b.** {1, 2, 3, 4, 6, 12}

4. Can every whole number be divided evenly by 1? Is 1 a factor of every whole number?

5. Find the set of all factors of each of the following.

a. 10 **b.** 15 **c.** 24 **d.** 18 **e.** 9 **f.** 14

6. Find the set of factors of each of the following.

a. 2 **b.** 13 **c.** 11 **d.** 23 **e.** 19 **f.** 7

Does 7 have any factors other than 1 and 7? Any whole number greater than 1 that has no factors other than 1 and the number itself is called a **prime number** (or a *prime*). Is 7 a prime number? Is 2 a prime number? is 10?

7. Any whole number greater than 1 that is not a prime number is called a **composite number**. Is 18 a composite number? is 20? is 19? is 0? is 1?

8. Make a table like this. Enter the name of each number from 2 to 50 in the proper column—*prime* or *composite*. List all the factors of the composite numbers, and place a star (*) next to the factors which are prime numbers.

Prime	Composite	Factors
2		
3		
	4	1, 2*, 4
5		
	6	1, 2*, 3*, 6

PRIME FACTORIZATION

1. Is this sentence true: $21 = 7 \times 3$? Are 7 and 3 prime numbers? Then the sentence shows 21 expressed as a product of primes, or the *prime factorization* of 21.

2. If we write $18 = 6 \times 3$, have we shown the prime factorization of 18? Why not?

Can 6 be expressed as 2×3? Then we can write the sentence below.

$$18 = 2 \times 3 \times 3$$

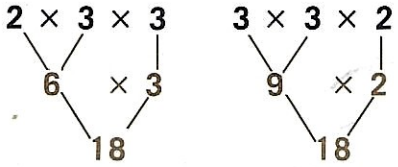
Does this sentence show the prime factorization of 18?

3. Does each of the following name a composite number? Express each as a product of primes.

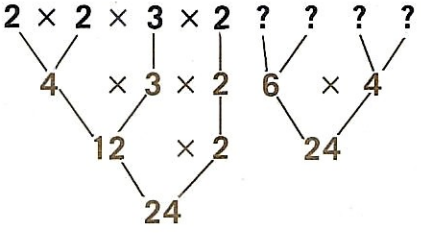
a. 20 **b.** 30 **c.** 16 **d.** 27 **e.** 40 **f.** 42

Do you think any composite number can be expressed as the product of primes?

4. Suppose in Exercise **2** we had begun by writing this sentence: $18 = 9 \times 2$. If we express 9 as a product of primes, do we then have the same prime factorization of 18 as before? (Disregard the order of the factors.) The **factor trees** at the right illustrate this.



5. At the right is a completed factor tree for 24 and one which you are to complete. Do the work. Apart from the order of the factors, are the prime factorizations of 24 the same?



▶ Every composite number has one prime factorization.

6. Show the prime factorization of each of the following.

a. 28 **b.** 27 **c.** 32 **d.** 36 **e.** 60 **f.** 72

1. Study the expression at the right. What number is named if N is replaced by 0? by 1? by 2? by 3?

$N \times 8$

Suppose you continued to replace N in the same way—by 4, by 5, by 6, and so on. Would you get this set of numbers?

$\{0, 8, 16, 24, 32, 40, \cdots\}$

This is the set of **multiples** of 8.

2. Using the same method, construct the set of multiples of 10.

$N \times 10$

3. Is this the set of multiples of 2? What is another way to describe this subset of the set of whole numbers?

$\{0, 2, 4, 6, 8, \cdots\}$

4. Now study sets A and B below.

$A = \{0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, \cdots\}$
$B = \{0, 6, 12, 18, 24, 30, 36, 42, \cdots\}$

How would you describe set A? set B?

Is 24 a multiple of 4? Is 24 also a multiple of 6? Then 24 is a **common multiple** of 4 and 6. Is 36 a common multiple of 4 and 6?

Is 40 a common multiple of 4 and 6? Why not?

Now look at set C. Is set C the set of common multiples of 4 and 6? Except for 0, the least number in the set is 12, and it is called the **least common multiple (l.c.m.)** of 4 and 6.

$C = \{0, 12, 24, 36, 48, \cdots\}$

5. Set X is the set of multiples of what number? Set Y is the set of multiples of what number? Using braces, show a listing of the set of common multiples of 8 and 10.

What is the least common multiple of 8 and 10?

$X = \{0, 8, 16, 24, 32, 40, 48, \cdots\}$
$Y = \{0, 10, 20, 30, 40, 50, 60, \cdots\}$

6. For each pair below, use braces and list the set of multiples for each number. Then list the set of common multiples of both numbers, showing at least three members of the set. Find the least common multiple for each pair of numbers.

a. 4, 3 **b.** 6, 9 **c.** 5, 7 **d.** 6, 3 **e.** 6, 10

7. What is the product of 8 and 10? Is 80 a common multiple of 8 and 10? Is the product of any two numbers a common multiple of both numbers? Is the product *always, sometimes,* or *never* the least common multiple?

8. Is 6 a multiple of 3? What is the least common multiple of 3 and 6?

What is the least common multiple of 4 and 8?

9. Suppose you wish to find the least common multiple of 12 and 10. Do you know that the number must be 12 or greater? Must the number also be a multiple of 12? Is 12 a multiple of 10? Then let us double 12, triple 12, and so on, until we find a number that is also a multiple of 10. Try 2×12; 3×12; 4×12; 5×12. Is 5×12, or 60, a common multiple of 10 and 12? Is 60 the least common multiple?

10. Find the least common multiple for each of the following.

a. 6, 8	**b.** 16, 12	**c.** 15, 10	**d.** 25, 15
e. 12, 18	**f.** 7, 14	**g.** 18, 4	**h.** 6, 16
i. 7, 9	**j.** 4, 12	**k.** 10, 2	**l.** 10, 4

11. We also speak of the common multiples of three or more numbers. Find two common multiples for each of the following.

a. 2, 3, 5 **b.** 4, 5, 10 **c.** 4, 6, 10 **d.** 3, 4, 5

12. Name the least common multiple for each of the sets of numbers named in **11a–d.**

If you make any mistakes, study the pages given at the right of the exercises.

1. Copy, and put $<$, $>$, or $=$ in place of $\square$ to make a true sentence. 2

a. $18 \square 17$ **b.** $29 \square \text{XXIX}$ **c.** $100 \square 99$

d. $804 \square 408$ **e.** $8412 \square 8421$ **f.** $4218 \square 4128$

2. Is $\{3, 4, 5\}$ a subset of $\{1, 2, 3, 4\}$? 3

3. Copy, and replace N with the decimal numeral that will make the sentence true. 7–8

a. $8 \times 10^2 = N$ **b.** $6 \times 10^3 = N$

4. Copy, and replace N with 10 or with a power of 10. 8–9

a. $900 = 9 \times N$ **b.** $7000 = 7 \times N$

5. Copy, and replace each ? with a power of 10, 10, or 1. 8–9

$$7{,}856{,}496 = (7 \times ?) + (8 \times ?) + (5 \times ?) + (6 \times ?) + (4 \times ?) + (9 \times ?) + (6 \times ?)$$

6. Copy, and replace N with a base ten numeral. 14–15

a. $1543_{six} = N$ **b.** $43_{six} = N$

7. Copy and add or subtract as the sign tells you. 16–21

a.
$$\begin{array}{r} \$87.50 \\ 4.35 \\ 9.88 \\ +\ 6.15 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 800 \\ 749 \\ 651 \\ +\ 300 \\ \hline \end{array}$$

c.
$$\begin{array}{r} \$8.75 \\ -\ 3.87 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 400 \\ -\ 197 \\ \hline \end{array}$$

8. Copy, multiply, and check. 23–31

a.
$$\begin{array}{r} \$1.35 \\ 25 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 465 \\ 231 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 738 \\ 40 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 432 \\ 600 \\ \hline \end{array}$$

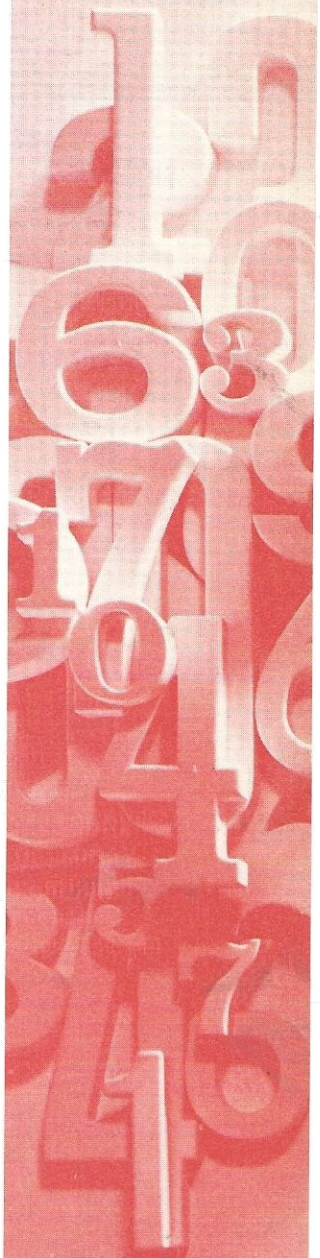
9. Copy, divide, and check. 32–41

a. $14\overline{)1246}$ **b.** $39\overline{)31{,}871}$ **c.** $41\overline{)31{,}871}$

10. Which of the following are prime numbers? Which are composite numbers? 43–44

a. 40 **b.** 37 **c.** 34 **d.** 31 **e.** 28

11. What is the least common multiple of 6 and 9? 45–46



EXTRA PRACTICE
306–308

DISCOVERING MORE ABOUT NUMBERS
334–336

MODERN MATHEMATICS THROUGH DISCOVERY

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